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Students' Construction of Conjectures Assisted by GeoGebra for Graphing Linear Equations: Cases of Female Students

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Abstract

This study aims to describe female students' conjecture construction on the topic of linear equation graphs with the assistance of GeoGebra. The study involved two female students who had mastered the necessary prerequisite material. Research instruments included conjecture construction tests and interviews, which were analyzed based on these conjecture construction indicators: 1) problem identification and exploration, 2) formulating conjectures, 3) testing and refining conjectures, and 4) proving conjectures. Results showed that, in the problem identification and exploration stage, students identified what was asked in the question, determined the information needed to answer it, and explored examples using GeoGebra. One student independently identified a pattern, while the other required an explanation. In the conjecture formulation stage, both students needed guidance to construct a general conjecture. Both students tested their conjectures, though only one needed to refine it. Initially, proof was conducted through examples; ultimately, both students succeeded in generalizing their proofs.

Keywords: *conjecture construction; linear equation; geogebra*

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INTRODUCTION

Reasoning and proof are core standards in mathematics instruction. In the curriculum goals of the Kurikulum Merdeka regarding the characteristics of mathematics learning, reasoning and proof are noted as essential elements within mathematics instruction (Kemendikbud, 2022). Similarly, the National Council of Teachers of Mathematics (NCTM) (2000) identifies five standards for mathematical learning processes, one of which is reasoning and proof. Reasoning is the activity, process, or thought sequence involved in making a hypothesis or conjecture based on observations, data, facts, or theories known or assumed to be true (Lailiyah et al., 2015). Reasoning relates to processes in identifying patterns, structures, or regularities to form and investigate conjectures, whereas proof pertains to the process of evaluating conjectures (NCTM, 2000). Based on this explanation, forming and investigating conjectures are integral parts of reasoning and proof activities.

A conjecture is a statement that appears plausible but has not yet been verified (Mason et al., 2010). According to Astawa et al (2018), a conjecture is a hypothetical mathematical statement, potentially true, and constructed by students using their knowledge based on given information or problems. Da Ponte (2001) also define a conjecture as a statement that serves as an answer to a question and is assumed to be true. Based on these definitions, a conjecture is a statement representing an answer to a question that has not been confirmed as true and is constructed based on prior knowledge.

Conjecture construction is the mental activity of forming hypotheses based on existing knowledge (Sutarto et al., 2018). This mental activity occurs in the mind and can be observed through students' behavior during problem-solving (Sutarto et al., 2016). Additionally, Zuraidha & Rosyidi (2022) state that the cognitive process of students when solving problems to create conjectures represents conjecture construction.

There are several stages in constructing conjectures. Da Ponte (2001) outlines the stages as 1) asking questions and making hypotheses, 2) testing and refining hypotheses, and 3) debating and proving hypotheses. Lerman & Zevenbergen (2006) also describe the stages of conjecture construction as: 1) exploring the problem to understand needs, 2) formulating and communicating hypotheses, 3) examining hypotheses and finding theoretical arguments to validate them, and 4) developing a proof accepted by mathematicians. Furthermore, (Astawa et al., 2018) identify five stages in conjecture construction: 1) understanding the problem, 2) exploring the problem, 3) formulating a hypothesis, 4) justifying the hypothesis, and 5) proving the hypothesis. In conjecturing, verifying the truth of hypotheses (justifying), and drawing conclusions from them (generalizing), students can utilize technology to achieve ideal outcomes (Putrawangsa & Hasanah, 2018). One technological tool that can effectively present mathematical content is GeoGebra.

GeoGebra is dynamic mathematics software for learning that includes geometry, algebra, and calculus (Hohenwarter & Hohenwarter, 2011). GeoGebra supports many mathematics topics and can be tailored to aid the learning process. It can serve as a tool for visualization, construction, and concept discovery (Isman, 2016), rendering abstract mathematical concepts more meaningful, allowing users to easily comprehend them. According to Rahmadi et al. (2015), using the GeoGebra application, students can learn to recognize geometric patterns.

Zuraidha & Rosyidi (2022) conducted a study on junior high students' conjecture construction on the topic of perimeter and area comparisons of rectangles. Their findings showed that subjects still struggled with formulating conjectures. Similarly, Azis & Rosyidi (2021) researched students' conjecture construction on open-ended classical analogy problems in quadratic functions, revealing that subjects struggled with justifying their conjectures. Furthermore, Hapizah et al (2020) investigated abstract mathematical conjectures in triangles and squares. Salmina & Nisa (2018) found gender differences in mathematical reasoning abilities, with female students outperforming male students.

According to the National Assessment Program, female students outperform males in reading, writing, speaking, and grammar, whereas males excel in arithmetic processes compared to females (Leder et al., 2014). In making conjectures, female students are able to articulate them more comprehensively (Manalu et al., 2020). Erawati & Purwati (2020) also concluded that females have stronger mathematical proof abilities compared to males.

Based on these studies, the researcher aims to further investigate how female students construct conjectures. Additionally, the researcher will explore conjecturing using GeoGebra, which has not been previously discussed. This study will focus on how female students construct conjectures on the topic of linear equation graphs with GeoGebra. The purpose of this research is to describe students' conjecture construction on the topic of

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linear equation graphs with GeoGebra, specifically among female students. This study is expected to provide insights into students' conjecture construction on the topic of linear equation graphs with the aid of GeoGebra, specifically for female students.

RESEARCH METHODOLOGY

This research is a qualitative study using a case study approach, designed to provide an in-depth description of the main issues. The case study focuses on female students. The selection of subjects was conducted using purposive sampling, based on the subjects' knowledge of linear equation graphs, as measured by a pre-test. The pre-test indicators for linear equation graph knowledge are presented in Table 1. The selected subjects were those who could correctly answer each item on the preliminary knowledge test.

Table 1. Pre-test Indicators for Linear Equation Graph Knowledge

Indicator	Item No.	Number of Items
Determining the position of a point in Cartesian coordinates	1	1
Identifying the quadrant in the Cartesian plane	2	1
Determining the position of a point in a quadrant of the Cartesian plane and its general form	3, 4, 5	3
Identifying points that a line passes through	6	1
Determining the intersection points of a line with the axes and their general form	7, 8	2
Identifying the linear equation and its general form	9, 10, 11, 12	4
Determining the general formula for the slope	13	1
Explaining the characteristics of a linear equation	14	1
Total		14

Data collection in this study was conducted using a task-based interview method. The task in question is a conjecture construction test, as shown in Figure 1 below.

There is a point A (1,0). What are the characteristics of a line passing through point A?

Figure 1. Conjecture Construction Test Task Used

Data analysis was performed by analyzing the interview results and student responses. Interviews were conducted based on conjecture construction indicators. These indicators, adapted from Zuraidha & Rosyidi (2022), are shown in Table 2.

Table 2. Conjecture Construction Indicators Used

Indicator	Code	Sub-Indicator
Identifying and exploring the problem	I1	Student identifies what is being asked
	I2	Student determines the information needed to solve the problem
	I3	Student explores relevant examples using GeoGebra
	I4	Student finds patterns based on examples
Formulating conjecture	R	Student creates a conjecture based on the identified patterns
Testing and refining conjecture	U1	Student tests the conjecture using other examples with GeoGebra
	U2	Student revises the conjecture if needed
Proving	B	Student proves the conjecture

conjecture

RESULTS AND DISCUSSION

The results of the study for the two selected subjects, ND and MY, are presented as follows.

Subject ND

The following includes the results of the conjecture construction task and the interview transcript with subject ND, presented at each stage.

Problem Identification and Exploration

The following excerpts are taken from ND's interview transcript in understanding the problem.

I : "What is being asked in the problem?"
ND101 : "The characteristics of a line that passes through the point (1,0)."
I : "What should you do first?"
ND102 : "Draw a line that passes through the point (1,0)."
I : "How do you create a line through the point (1,0)?"
ND103 : "Make another point or determine its slope."
I : "Good, now go ahead and draw a line and explore its characteristics using the Geogebra application."
ND104 : "Okay" (the student explores using the Geogebra application).
I : "How are you using Geogebra to explore?"
ND105 : "I input the equation of the line that passes through the point (1,0) and slope 5, then I change the slope."
I : "When you change the slope, what do you observe?"
ND106 : "The line intersects the y-axis at a certain point."
I : "From your exploration, did you find a pattern?"
ND107 : "Yes, I found a pattern: if the slope is 2, it intersects the y-axis at $y = -2$; if the slope is 10, it intersects at $y = -10$; if the slope is -5, it intersects at $y = 5$; and if the slope is -6, it intersects at $y = 6$."

ND understood the question being asked, which is to find the characteristics of a line passing through the point (1,0) [ND101]. ND also understood that he needed to draw a line through this point by creating another point or determining the slope [ND102 and ND103]. ND then explored using Geogebra [ND104]. ND explained the exploration process, initially creating the line and then varying the slope to observe its characteristics [ND105 and ND106]. After the exploration, ND was able to identify a pattern in the characteristics of lines passing through the point (1,0): if the slope is 2, the line intersects the y-axis at $y = -2$; if the slope is -5, it intersects at $y = 5$, and so on [ND107].

Conjecture Formulating

The conjecture formulated by subject ND is presented in Figure 2. In Figure 2, the subject makes a conjecture about the gradient, stating:

- If the gradient is positive, it will intersect the y-axis at a negative point.
- If the gradient is negative, it will intersect the y-axis at a positive point.

The subject provides examples for this conjecture with gradient values of 5 and -7.

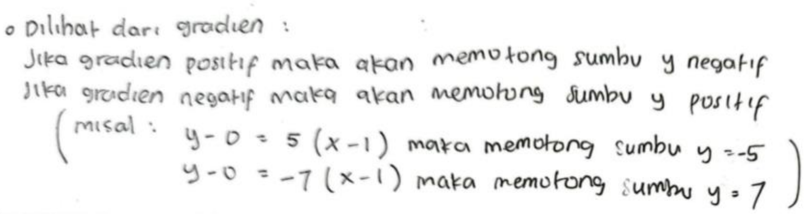

Figure 2. Conjecture Formulation by Subject ND

The following is the interview transcript with ND at the stage of formulating the conjecture.

I : "Generally speaking, based on the pattern you observed, what is your hypothesis about the characteristics of a line that passes through the point (1,0)?"

ND201 : "What do you mean by 'generally speaking'?"

I : "So, you previously provided examples, but those only apply to certain slopes. Here, there are many slopes that follow the same pattern as the examples you gave."

ND202 : "Then, do I need to write out all the numbers?"

I : "You can generalize it by referring to positive or negative slopes."

ND203 : "Oh, I understand. So, if the slope is positive, the line will intersect the *y*-axis at a negative point. If the slope is negative, it will intersect the *y*-axis at a positive point."

Initially, ND could not formulate a general conjecture and did not know how to articulate it [ND201 and ND202]. After receiving an explanation, ND was able to formulate the conjecture shown in Figure 2: the characteristics of a line passing through the point (1,0), based on its slope, are that if the slope is positive, the line will intersect the y-axis at a negative point; if the slope is negative, it will intersect the *y*-axis at a positive point [ND203, Figure 2].

Testing and Refining the Conjecture

In testing and refining the conjecture, the subject used Geogebra to aid their investigation. Figure 3 shows a screenshot of one of the subject's activities with GeoGebra for testing and refining the conjecture using examples of positive and negative gradient values.

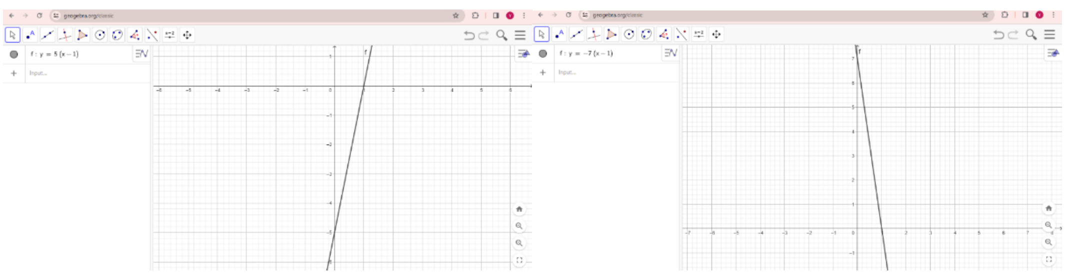

Figure 3. Testing Results of Subject ND with Slopes of 5 and -7

The following is the interview transcript with ND during the testing and refining stage of the conjecture.

P : "Try to check if your hypothesis holds for all slopes."

ND301 : "Yes, it does. For example, if the slope is 5, it intersects the *y*-axis at $y = -5$. If the slope is -7, it intersects the *y*-axis at $y = 7$."

P : "So, is it correct that if the slope is positive, it will intersect the y-axis at a negative point, and if the slope is negative, it will intersect at a positive point? Or is there anything you think needs to be revised?"

ND302 : "Yes, it's correct. No revision is needed."

ND tested the conjecture with other examples, using slopes of 5 and -7 with the help of Geogebra [Figure 3]. The results aligned with the conjecture, showing intersections at $y = -5$ and $y = 7$, respectively [ND301]. Therefore, ND felt no need to refine the conjecture [ND302].

Proving the Conjecture

The proof of the conjecture by subject ND is shown in Figure 4.

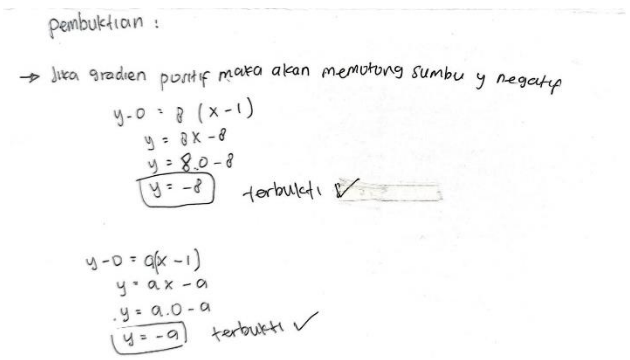

Figure 4. Conjecture Proof by Subject ND

The following is the interview transcript with ND at the stage of proving the conjecture.

I : "Prove one of the hypotheses you made!"

ND401 : "How do I prove it?"

I : "Proving means showing that your hypothesis holds in general (gives an example of proving a conjecture for square numbers)."

ND402 : "Oh, I understand."

I : "So, which hypothesis will you prove?"

ND403 : "I will prove the hypothesis: If the slope is positive, the line will intersect the y-axis at a negative point."

I : "Yes, go ahead."

ND404 : "The equation of a line using the slope is $y - y_1 = m(x - x_1)$. Since the point is (1,0), the equation becomes $y - 0 = m(x - 1)$. I'll assume the slope is 8. So $y - 0 = 8(x - 1)$; $y = 8x - 8$. What do I do next?"

I : "Your conjecture states that if the slope is positive, it will intersect the y-axis at a negative point. What is the general form of a point of intersection on the negative y-axis?"

ND405 : "$(0, -b)$."

I : "You can substitute $x = 0$ into your equation!"

ND406 : "Does it become $y = 8(0) - 8$?"

I : "Yes, correct."

ND407 : "So $y = -8$."

I : "Earlier, you set $x = 0$ and obtained $y = -8$. What does that

mean about the point?"
ND408 : "It means this point intersects the negative y-axis."
I : "Yes, correct. So, what's the conclusion?"
ND409 : "The statement I made is correct."
I : "Good. However, you only proved it for a slope of 8. Does your statement apply to all positive slopes?"
ND410 : "Yes, it does because when the slope is multiplied by 0, it always results in 0. So, y will always be negative."
I : "Now try proving it generally."
ND411 : "Oh, so $y - 0 = a(x - 1)$; $y = ax - a$; for $x = 0$, so $y = a(0) - a$; $y = -a$. It's proven. So, the hypothesis that if the slope is positive, it will intersect the y-axis at a negative point is proven."

Based on Figure 2 and the interview transcript, ND initially did not know how to prove the conjecture [ND401]. After being given an example of proving a conjecture with square numbers, ND understood [ND402]. ND then chose to prove one of his conjectures, "If the slope is positive, the line will intersect the y-axis at a negative point" [ND403]. In the process of proving, ND became confused about the next steps [ND404]. Due to the confusion, ND received guidance through several steps [ND405, ND406, ND407, and ND408]. Afterward, ND was able to conclude that his proof was correct [ND409 and Figure 4]. However, ND's initial proof did not apply universally, as it only proved the conjecture for a single slope value. ND realized that his conjecture holds in general because any value multiplied by 0 yields 0 [ND410]. Thus, ND re-proved the conjecture for a general slope value, aaa, following the same approach [ND411, Figure 3], demonstrating that his conjecture was universally valid.

Subject MY
Below are the results of the conjecture construction task and the interview transcript of subject MY, presented at each stage.
Problem Identification and Exploration
The following excerpt is a transcript from the interview with subject MY in understanding the problem.

I : "What is the question asking?"
MY101 : "Characteristics of the line."
I : "Characteristics of which line?"
MY102 : "Characteristics of the line passing through the point (1,0)."
I : "What should you do first?"
MY103 : "Draw a line through the point (1,0)."
I : "How do you make a line through the point (1,0)?"
MY104 : "Create another point."
I : "Is that all?"
MY105 : "Or by determining the gradient."
I : "Good, now go ahead and create the line and explore its characteristics using the GeoGebra application."
MY106 : "Alright, ma'am." (The student explores using the GeoGebra application.)
I : "How are you using GeoGebra to explore?"
MY107 : "Um, just like what you showed earlier."
I : "What do you always change when using GeoGebra?"

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MY108 : "The location of this point."
I : "Then what do you observe?"
MY109 : "Um, I'm not sure, ma'am."
I : "You can look at the gradient value from the equation or see which axis the line intersects."
MY110 : "Oh, yes, I'll observe the gradient value from the equation or see which axis the line intersects."
I : "From your exploration, did you find any patterns?"
MY111 : "What do you mean by that, ma'am?"
I : "Maybe if you move the other point in this quadrant (while showing on GeoGebra), you will always see where the line intersects, or you can observe changes in the equation, or the gradient."
MY112 : "I'm still confused, ma'am. Can you give an example?"
I : "Let me show you, for example, a line through the point (4,0). I'll add another point to make the line, say point B. If point B is (-4,4), the gradient is -0.5. If point B is (-6,1), the gradient is -0.1."
MY113 : "Oh, I understand. So, there's a pattern for the characteristics of a line passing through the point (1,0). If point B is (-6,-2), the gradient is 0.29. If point B is (-8,-6), the gradient is 0.67."

MY identified what the question was asking, which was the characteristics of the line [MY101]. However, the answer was incomplete as it didn't mention the line passing through a specific point. Thus, the researcher asked again, and MY completed the answer, mentioning the characteristics of the line through the point (1,0) [MY102]. MY knew the initial step, which was to first draw the line through the point (1,0) [MY103]. MY understood the knowledge needed to create the line by making another point [MY104]. This knowledge was not fully articulated, but when prompted, MY was able to complete it, noting that in addition to creating another point, the gradient could also be determined [MY105]. MY explored examples using GeoGebra [MY106]. MY was confused when asked to explain the exploration process and just followed the examples demonstrated by the researcher earlier [MY107]. With some guiding questions, MY could explain the exploration process [MY108, MY109, and MY110]. When asked to find a pattern, MY felt confused [MY111 and MY112]. After further explanation and examples, MY was able to identify a pattern for the characteristics of the line passing through the point (1,0) [MY113].

Conjecture Formulating

The conjecture formulated by subject ND is presented in Figure 5. In Figure 5, the subject makes a conjecture about the gradient, stating:

- If point B is in quadrant III, then the gradient is below 1.
- If point B is in quadrant II, then the line intersects the positive x-axis.
- If point B is in quadrant II, then the gradient is negative.
- If point B is in quadrant I, then the gradient is greater than 1.

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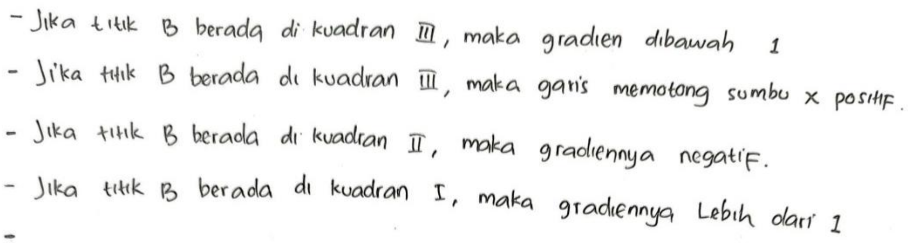

Figure 5. Conjecture Formulation by Subject MY

Below is the transcript of the interview conducted with MY during the conjecture formulation stage.

I	: "Generally speaking, based on the pattern you observed, what is your conjecture regarding the characteristics of the line passing through the point (1,0)?"
MY201	: "What do you mean by generally speaking?"
I	: "So, the examples you gave only apply to a few points. However, here there are many points that make the line have a gradient that may match the pattern you mentioned."
MY202	: "What do you mean, ma'am? I still don't understand."
I	: "For example, for the line I created earlier with the point (4,0), I added another point to make the line, say point B. If point B is (-4,4), the gradient is -0.5. If point B is (-6,1), the gradient is -0.1. If point B is (-7,10), the gradient is -0.91. So, I can make a general conjecture that if point B is in quadrant II, then the gradient is less than 0."
MY203	: "So, my conjecture would be: 1) If point B is in quadrant III, then the gradient is below 1; 2) If point B is in quadrant III, then the line intersects the positive x-axis; 3) If point B is in quadrant II, then the gradient is negative; 4) If point B is in quadrant I, then the gradient is greater than 1."

Initially, MY could not formulate a general conjecture and did not know how to create one [MY201 and MY202]. After being given an example, MY was able to formulate four conjectures shown in Figure 5 regarding the characteristics of the line passing through the point (1,0): 1) If point B is in quadrant III, the gradient is below 1; 2) If point B is in quadrant III, the line intersects the positive x-axis; 3) If point B is in quadrant II, the gradient is negative; 4) If point B is in quadrant I, the gradient is greater than 1 [Figure 5].

Testing and Refining the Conjecture

In testing and refining the conjecture, the subject used GeoGebra to support their investigation. Figure 6 shows a screenshot of one of the subject's activities with GeoGebra for testing and refining the conjecture using examples of points (-2, -1) and (-2, -8).

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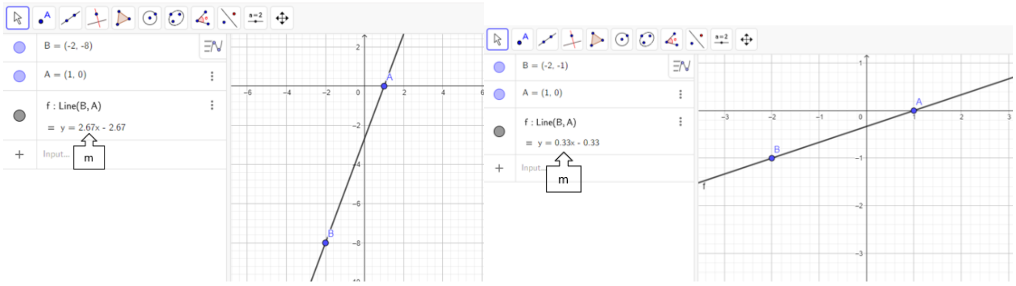

Figure 6. MY's Testing Results with Points (-2,-1) and (-2,-8)

The following is the transcript of the interview conducted with MY during the conjecture testing and refining stage.

I	: "Try checking your first conjecture, does it apply to all points in quadrant III?"
MY301	: "Yes, it applies. If point B is (-2,-1), the gradient is 0.33 (testing results are in Figure 6)."
I	: "What about if point B is (-2,-8)? What is the gradient? Does your first conjecture also apply in this case?"
MY302	: "If point B is (-2,-8), the gradient is 2.67. Oh, it doesn't apply. So, my conjecture is incorrect (testing results are in Figure 7)."
I	: "So how can you make your conjecture accurate?"
MY303	: "But in some parts, my conjecture does apply, while in others, it doesn't."
I	: "What should you adjust to make your conjecture generally applicable?"
MY304	: "I don't know, ma'am."
I	: "You could add conditions to your conjecture."
MY305	: "Oh, so I need to add boundary conditions for my conjecture to apply generally."

MY tested the conjecture with additional examples using GeoGebra [Figure 6]. Initially, MY was confident that the conjecture applied generally, testing the first conjecture: if point B is in quadrant III, then the gradient is below 1 [Figure 5]. MY tested it with point (-2,-1) and found the gradient below 1, specifically 0.33 [MY301]. However, MY discovered that the conjecture does not apply generally [MY302 and MY303], requiring refinement. MY, who was unsure how to refine the conjecture, received assistance [MY304]. As a result, MY improved the conjecture by adding a condition [MY305].

Proving the Conjecture

The conjecture proof by subject MY is shown in the following Picture 7.

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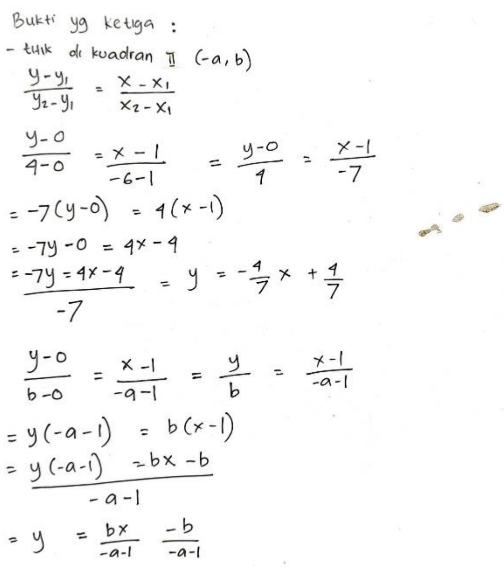

Figure 7. MY's Conjecture Proof

The following is the interview transcript with MY during the conjecture proof stage.

I : "Try proving your third conjecture. Is it true that if point B is in quadrant II, then the gradient is negative?"

MY401 : "How do I prove it?"

I : "Proving means showing that your conjecture holds generally (provides an example of proving a quadratic number conjecture)."

MY402 : "So, what should I do first?"

I : "You can prove it by using the line equation between two points!"

MY403 : "So, $\frac{y-y1}{y2-y1} = \frac{x-x1}{x2-x1}$. Point ND(1,0) and point B, say (-6,-4). Thus, $\frac{y-0}{4-0} = \frac{x-1}{-6-1}$; $\frac{y-0}{4} = \frac{x-1}{-7}$; $-7(y-0) = 4(x-1)$; $-7y = 4x-4$; $y = \frac{-4}{7}x + \frac{4}{7}$. What's next?"

I : "Now, can you see the gradient of that line? Is it negative or positive?"

MY404 : "-4/7, negative."

I : "So, does the gradient match your third conjecture?"

MY405 : "Yes, it matches."

I : "Then we can conclude that your conjecture is correct. But does it apply to all points in quadrant II?"

MY406 : "I don't know, ma'am."

I : "Then you'll need to prove it for all points in quadrant II!"

MY407 : "How do I do that, ma'am?"

I : "You can generalize it by using letters. What is the general form of a point in quadrant II?"

MY408 : "$(-a, b)$."

I : "Now you can replace point B with $(-a, b)$."

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MY409 : "So, $\frac{y-y1}{y2-y1} = \frac{x-x1}{x2-x1}$. Point ND(1,0) and point B, say $(-a, b)$. Thus, $\frac{y-0}{b-0} = \frac{x-1}{-a-1}$; $\frac{y}{b} = \frac{x-1}{-a-1}$; $y(-a-1) = b(x-1)$. What's next?"

I : "You can divide both sides by $(-a-1)$."

MY410 : "$\frac{y(-a-1)}{(-a-1)} = \frac{bx-b}{(-a-1)}$; $y = \frac{bx}{(-a-1)} - \frac{b}{(-a-1)}$."

I : "What is the gradient of that equation?"

MY411 : "The gradient is $b/(-a-1)$, so it's negative."

I : "Yes, that's correct. So what's the conclusion?"

MY412 : "The conjecture I made is correct. If point B is in quadrant II, then the gradient is negative."

The conjecture to be proven by MY was the third conjecture: if point B is in quadrant II, then the gradient is negative [Figure 5]. Initially, MY did not know how to prove the conjecture [MY401], so the researcher provided an example of proving a conjecture about quadratic numbers. Even so, MY was still confused about the initial steps [MY402]. With assistance, MY completed the proof's first steps [MY403 and MY404], concluding that the proof was correct [MY405 and Figure 7]. However, MY's proof was not general, as it only applied to one point. Consequently, MY worked to prove the conjecture for all points in quadrant II. MY, initially unsure of how to proceed [MY406 and MY407], generalized the proof using a point in quadrant II, (-a, b) [MY408]. With some help, MY completed the proof, showing that the conjecture was generally correct [MY412 and Figure 7].

Based on the research findings, the flowchart of students' conjecture construction on the topic of linear equation graphs using GeoGebra for female students is presented as follows.

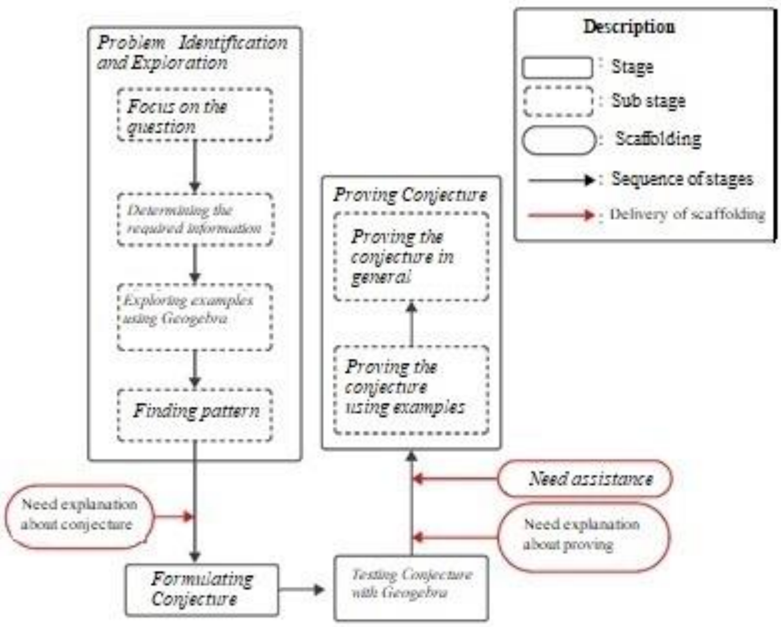

Figure 8. ND's Conjecture Construction Flowchart

https://dx.doi.org/10.26594/jmpm.v9i2.4858

Based on Figure 8, ND did not understand how to formulate a general conjecture, thus requiring an explanation on how to formulate a conjecture generally. After testing the conjecture, ND found no errors in her conjecture, so no revisions were necessary. Another difficulty ND encountered was in proving the conjecture, for which she needed explanation and assistance. Initially, ND proved the conjecture using an example. However, she later succeeded in proving it generally.

The construction of the conjecture by MY was similar to ND's, but MY required more guidance at several stages of conjecture construction.

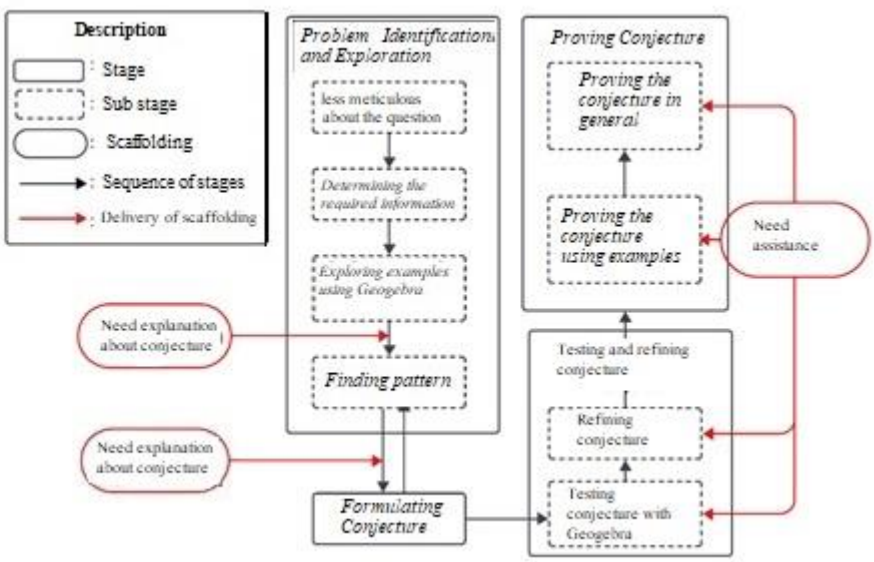

Figure 9. MY's Conjecture Construction Flowchart

Based on Figure 9, MY was less meticulous about the question asked. MY only focused on the characteristics of the line, whereas the problem required characteristics of the line passing through point (1,0). In identifying patterns, MY needed an explanation as he did not understand the meaning of the formed pattern. Additionally, in formulating the conjecture, MY didn't understand how to make a hypothesis, so an explanation and examples were provided. MY then identified the pattern and formulated four conjectures. When testing the conjecture, MY initially found no errors. After receiving assistance, MY recognized errors in his conjecture and revised it. Like ND, MY initially proved the conjecture using examples with assistance but was later able to prove it generally.

In the identification and problem exploration stage, there are four key steps students must undertake: determining what is being asked, identifying required information, exploring, and identifying patterns. In determining what was being asked, one student identified it correctly, but another lacked accuracy, which could lead to errors in solution steps. This aligns with research (Saparwadi, 2022) indicating that a common error in understanding mathematics is students' lack of accuracy in reading and understanding questions. Secondly, students identified the information needed to solve the question. In this step, both students identified the necessary information for the initial steps of conjecture construction, starting with plotting a line passing through point (1,0). Prior knowledge is crucial as an initial strategy for solving the question (Yuwono et al., 2018).

In exploring examples, students used GeoGebra to explore other examples, observing the characteristics of lines passing through point (1,0). One student did not thoroughly explore all examples, trying only a few cases. This is similar to findings by Rosyidi et al. (2024), which stated that students often do not consider all possible cases. In identifying patterns based on examples, one student identified patterns from the exploration, while the other required assistance in recognizing the pattern. Observing patterns is one form of mathematical thinking habit (Handayani, 2015), and students' patterns differ, as not all students are accustomed to recognizing them.

In the conjecture formulation stage, students should be able to construct conjectures based on observed patterns. Initially, neither student could create a conjecture. They required explanations on how to form a general conjecture, with both eventually producing general conjectures based on observed patterns. One student needed an example of formulating a general conjecture. This indicates that algebraic concepts, number properties, and Cartesian coordinate systems play a significant role in constructing conjectures on this topic. It also suggests that students require more training in conjecture formulation, as indicated by low conjecturing skills (Cahya & Warmi, 2019).

In testing and refining conjectures, students used GeoGebra to simplify conjecture testing, entering gradients or other points to test their conjectures. One student accurately tested the conjecture using different gradients without needing revisions, while the other needed assistance. After testing, a conjecture required adjustments due to inaccuracies. With GeoGebra, students quickly identified if their conjecture did not hold under other conditions, as GeoGebra enables efficient and accurate object construction (Asngari, 2015). Students also required guidance to refine incorrect conjectures to achieve a valid conclusion (Oktavia et al., 2019), ensuring the conjecture applies generally.

In the conjecture-proofing stage, students were initially unable to proceed because they didn't know how to prove the conjecture. They were given explanations and examples of proofs. Additionally, they needed help with specific proof steps, struggling with identifying line equations and selecting substitution points. This aligns with findings by Oktavia et al. (2019), which highlighted students' struggles with basic algebra, solution steps, and proper substitution in linear equation graphing. Since students were unfamiliar with proof questions, they initially used examples to validate their conjectures. However, when prompted, they were able to prove them deductively.

Thus, students must master algebraic topics thoroughly. Although the topic involves linear equation graphs, several conjecture construction steps require algebra for solutions. Students should also be skilled in GeoGebra. Beyond this study, GeoGebra is valuable for visualizing mathematics, particularly in geometry.

Overall, female students showed low interest in forming hypotheses about the given task. This corresponds with Kartono's findings in (Narpila, 2019), where female students generally have limited interest in theoretical problems, showing a preference for practical over theoretical tasks. In several stages of conjecture construction, female students required examples to complete a stage, struggling with abstract reasoning from verbal explanations alone, necessitating examples for better understanding.

Students generally encountered difficulties in constructing conjectures, particularly in formulating and proving them. Both students were unsure how to generalize a conjecture, with one needing an example. Proofing was also challenging, requiring guidance in identifying initial steps. Both initially proved conjectures using examples, but with guidance, they were able to complete deductive proofs.

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CONCLUSION AND SUGGESTIONS

Based on the results and discussion, there are two out of four stages of conjecture construction that female students still find challenging: formulating and proving conjectures. In general, the process of conjecture construction on the topic of linear equation graphs among female students can be summarized at each stage.

In the problem identification and exploration stage, students identify what the question asks, determine the information needed to answer it, explore examples using GeoGebra, and one student independently finds the pattern in the linear equation graph, while the other requires guidance to do so. In the conjecture formulation stage, both students need guidance on how to formulate a general conjecture, with one student requiring an example for further clarity. In the testing and refining stage, students test the conjecture using different gradients or points, with one student refining her conjecture. In the proving stage, both students need assistance. Initially, they use numerical examples to prove the conjecture, but after being prompted to prove it generally, they manage to do so.

Based on the findings, the researcher suggests that students should become accustomed to making and proving conjectures using GeoGebra, as these aspects present challenges for students. In this study, students also engaged frequently with the researcher, which helped them construct conjectures even though they had no prior experience with it. The researcher recommends investigating conjecture construction through a collaborative model, allowing students to discuss within their groups.

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